Recursion

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1 Introduction

Recursion has its own section in the ACSL short-answer contest, and is embedded in many competitive programming problems. This lecture is a guide to recursion problems you will see on all programming contests. Formally defined, a recursive function is one that calls itself until a base case is satisfied.

2 Recursive Functions

1. Find f(-1):

$$f(x) = \begin{cases} x - f(x+1) & \text{if } x < 3 \\ f(2x) & \text{if } 3 <= x < 5 \\ x + 1 & otherwise \end{cases}$$

2. Find f(7,4):

$$f(x) = \begin{cases} x - y & \text{if } x <= 0 \\ x - f(x-2, y-1) & \text{if } x > 0 \text{ and x is even} \\ y - f(y-2, x-1) & \text{if } x > 0 \text{ and x is odd} \end{cases}$$

2. Find f(4):

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = 1 \\ f(x-1) + f(x-2) & \text{otherwise} \end{cases}$$

3 Problems

1. A square carpet is designed by starting with a 27 x 27 square and dividing it into 9 sections, coloring the 5 square-sections on the diagonals and subdividing the others into 9 sections as above. When the subdivided squares are each 1 x 1 and the diagonals are colored, how many 1 x 1 squares are uncolored?

2. Find f(f(f(1, 0), 1), 0) given:

$$f(x) = \begin{cases} f(x-2, y+1) + 3 & \text{if } x > y \\ f(x+1, y-3) - 4 & \text{if } x = y \\ x + 2y & \text{if } x < y \end{cases}$$

3. (UVa Online Judge, 624 - CD) You have a tape recorder with a capacity of N minutes. Additionally, we are given a collection of songs on a CD and their associated lengths. Figure out how to choose tracks from CD to put on the tape recorder to get most out of tape space and have as short unused space as possible. No track is longer than N minutes, and all song lengths (and N) are integers.

4. Review and implement the recursive complete search problems given last week.