Recursion Problems

6 October 2017

1 Recursive Problems

Problem 1. Find j(15). Given:

$$j(x) = \begin{cases} 2[j(x-3)+2] & \text{if } x >= 10 \\ j(x-2)+3 & \text{if } 8 <= x < 10 \\ x & \text{if } x < 8 \end{cases}$$

Problem 2. Find f(10). Given:

$$f(x) = \begin{cases} f(f(x-2))+1 & \text{if } x > 1 \\ 1 & \text{if } x = 0 \\ 0 & \text{if } x = -1 \end{cases}$$

Problem 3. Find f(16, 8). Given:

$$f(x,y) = \begin{cases} f(x-y, y-1)+2 & \text{if } x > y \\ x-1 & \text{if } x = y \\ x+y & \text{if } x < y \end{cases}$$

Problem 4. Evaluate f(22) given the following:

$$f(x) = \begin{cases} x + f(x+3) & \text{if x is prime and larger than 2} \\ \frac{x}{2} * f(x-5) & \text{if x is even} \\ x+1 & \text{otherwise} \end{cases}$$

(Source: ACSL All-Stars 96-97)

Problem 5. Find f(9, 4). Given:

$$f(x) = \begin{cases} f(x-2, 2y) & \text{if } x > 6 \\ f(x-3, y-1)+2 & \text{if } 2 \leq \text{x} \leq 6 \\ 2x-y & \text{otherwise} \end{cases}$$

(Source: ACSL Senior 2001)

Problem 6. Given:

$$f(x) = \begin{cases} x/2 & \text{if x is even} \\ f(f(3x+1)) & \text{otherwise} \end{cases}$$

if $x = 2^k + 1$. Then, what is f(x).

Problem 7. Ackerman's function is defined as:

$$A(x,y) = \begin{cases} y+1 & \text{if } x = 0 \\ A(x-1,1) & \text{if } x \neq 0 \text{ and } y = 0 \\ A(x-1,A(x,y-1)) & \text{if } x \neq 0 \text{ and } y \neq 0 \end{cases}$$

Find A(3, 4).

Problem 8. Find f(f(f(50))) where [a/b] = greatest integer function and = absolute value function

$$f(x) = \begin{cases} f([x/3]) + 2 & \text{if } x > 20 \\ f(|10 - x|) - 3 & \text{if } 8 \leq x \leq 20 \\ \mathrm{x}^2 - x + 1 & \text{if } x < 8 \end{cases}$$

Problem 9. Find F(12,3) given that odd and even are defined over the set of integers.

$$f(x) = \begin{cases} F(X/2, Y-1) + 3X & \text{if X is even and Y is odd} \\ F(Y-2, X+1) + X & \text{if X is even and Y is odd} \\ \mathrm{X}^2 - Y^2 & \text{if X is odd and Y is even} \\ F(Y+1, X-1) – 2Y & \text{if X is odd and Y is odd} \end{cases}$$

Challenge Code Problem for the Bored Every day, Farmer John walks through his pasture to check on the well-being of each of his cows. On his farm he has two breeds of cows, Holsteins and Guernseys. His HH Holsteins are conveniently numbered 1...H1...H, and his GG Guernseys are conveniently numbered 1...G1...G (1H1000,1G10001H1000,1G1000). Each cow is located at a point in the 2D plane (not necessarily distinct).

Farmer John starts his tour at Holstein 1, and ends at Holstein HH. He wants to visit each cow along the way, and for convenience in maintaining his checklist of cows visited so far, he wants to visit the Holsteins and Guernseys in the order in which they are numbered. In the sequence of all H+GH+G cows he visits, the Holsteins numbered 1...H1...H should appear as a (not necessarily contiguous) subsequence, and likewise for the Guernseys. Otherwise stated, the sequence of all H+GH+G cows should be formed by interleaving the list of Holsteins numbered 1...H1...H with the list of Guernseys numbered 1...G1...G.

When FJ moves from one cow to another cow traveling a distance of DD, he expends D2D2 energy. Please help him determine the minimum amount of energy required to visit all his cows according to a tour as described above.

(Source: USACO Gold)