American Computer Science League

2017-2018 **Contest #4**

SENIOR DIVISION SOLUTIONS

1. Graph Theory

To find the number of paths of length 3 from, each vertex, add the entries in the cubed adjacency matrix in each row. B has the most at 32.

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}^3 = \begin{vmatrix} 5 & 8 & 3 & 6 & 6 \\ 6 & 10 & 3 & 8 & 5 \\ 4 & 5 & 2 & 3 & 3 \\ 5 & 9 & 3 & 7 & 6 \\ 3 & 6 & 2 & 5 & 5 \end{vmatrix}$$

1. B 32

2. Graph Theory

The 13 cycles from B are: BAB, BAFB, BCB, BCDEB, BCDEAB, BCDEAFB, BCDEFB, BCFB, BDEB, BDEAB, BDEAFB, BDEFB, BFB

2. 13

3. Digital Electronics

The circuit translates to: $\overline{(\overline{A}(\overline{A+B}))(\overline{BC+\overline{C}})}$

$$\overline{(\overline{A}(\overline{A+B}))(\overline{BC+\overline{C}})} = \overline{\overline{A}(\overline{A+B})} + \overline{\overline{(BC+\overline{C})}} = \overline{\overline{A}} + \overline{\overline{A+B}} + BC + \overline{C}$$
$$= A + A + B + BC + \overline{C} = A + B(1+C) + \overline{C} = A + B + \overline{C}$$

So $A + B + \overline{C} = 0$ implies $A = 0 \wedge B = 0 \wedge \overline{C} = 0$. (0, 0, 1) makes it FALSE.

3. (0, 0, 1)

4. Digital Electronics

The circuit translates to: $A(\overline{A+B} \oplus B\overline{C}) + \overline{C}$

$$A(\overline{A+B} \oplus B\overline{C}) + \overline{C} = A(\overline{\overline{A+B}B\overline{C}} + \overline{A+B}\,\overline{B\overline{C}}) + \overline{C}$$
$$= A((A+B)B\overline{C} + \overline{A}\,\overline{B}(\overline{B} + \overline{\overline{C}})) + \overline{C}$$
$$= A(AB\overline{C} + B\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{B}C) + \overline{C}$$
$$= AB\overline{C} + AB\overline{C} + A\overline{A}\,\overline{B} + A\overline{A}\,\overline{B}C + \overline{C}$$
$$= AB\overline{C} + \overline{C} = \overline{C}(AB+1) = \overline{C}$$

4. $\overline{C}$

5. Assembly Language

This program converts 7325_8 to a base 10 number.

5. 3797